Illumination Angle Compensation in Kirchhoff Depth Migration

Walter Kessinger, Geotrace Technologies*

Summary

Migration/inversion theory implies that illumination angle compensation should be accounted for in Kirchhoff prestack depth migration, although it is usually neglected. Comparisons of subsalt Kirchhoff migration results with and without illumination angle compensation demonstrate that inclusion of this amplitude weighting term can dramatically improve the migrated image, justifying the increase in implementation effort and cost.

Introduction

It is generally acknowledged that wave equation prestack depth migration is inherently superior to Kirchhoff prestack depth migration for severe depth imaging problems such as beneath salt (Roden and Gochioco, 2002). The inferior performance of Kirchhoff depth migration is usually attributed to three factors:

1. Difficulty in accurately ray tracing through complex velocity models;
2. The failure of many Kirchhoff implementations to include complete multipathing solutions; and
3. Incomplete implementation of amplitude terms in Kirchhoff migrations.

It is worth noting that these criticisms are of Kirchhoff migration implementations, not of the underlying Kirchhoff migration theory.

An extensive body of work exists analyzing Kirchhoff prestack depth migration in terms of migration/inversion theory (for example, Miller et al., 1987; Jin et al., 1992). These treatments demonstrate that the theoretical foundations underlying Kirchhoff migration are similar to those underlying so-called wave equation migration methods. However, there are fundamental differences in the positioning and imaging mechanisms that each method employs.

Because of intrinsic differences in the formulation and implementation of Kirchhoff and wave equation prestack depth migrations, each migration type has different strengths and weaknesses. A commonly acknowledged strength of wave equation methods is their natural and inherent incorporation of amplitude factors and multipathing solutions. Kirchhoff methods, on the other hand, have inherent practical advantages because of their extreme flexibility in producing subsets of the output image volume. This flexibility extends to a wide range of options for segregating output images into common image gathers, a property which is particularly important for purposes of iterative velocity analysis and AVO analysis.

Because computational and other practical considerations will continue to favor Kirchhoff migration methods for many applications in the near future, it is desirable to examine the flaws responsible for quality problems in Kirchhoff imaging and determine whether better implementation of existing theory can improve Kirchhoff depth migration results.

My focus here is on the common neglect of amplitude terms that compensate for irregular illumination at the image point. When complex velocity variations in overlying structure create an "illumination footprint" at the target depth, failure to compensate for this illumination irregularity results in the generation of spurious events, frequently with steep dips, that have earned the label "Kirchhoff artifacts." These artifacts can degrade the final migration results and have adverse effects on interpretation.

Theory

Schleicher et al. (1993) defined prestack Kirchhoff migration as a weighted diffraction stack that can be expressed in the form

$$I(\vec{x}) = \frac{-1}{2\pi}\int_A d\vec{\xi}\;\; w_Q(\vec{\xi},\vec{x})\frac{d}{dt}(U(\vec{\xi},\tau_D(\vec{\xi},\vec{x}))), \quad (1)$$

where the image point $I(\mathbf{x})$ is formed by a weighted summation over a travel time surface through an ensemble of seismic data $U(\boldsymbol{\xi},\tau_D)$. The acquisition geometry is denoted by $\boldsymbol{\xi}$,, and τ_D is the travel time for reflections from an image point at $\mathbf{x}$.

For a constant-azimuth common-offset ensemble, $\boldsymbol{\xi} = (\xi_1, \xi_2)^T$ denotes the midpoint location of each trace. (For a non-redundant ensemble, traces can be uniquely identified by a single pair of coordinates.) The travel time surface $\tau_D(\boldsymbol{\xi},\mathbf{x})$ is the sum of the travel times from the source to the image point and from the image point to the receiver:

$$\tau_D(\vec{\xi},\vec{x}) = \tau(\vec{s},\vec{x}) + \tau(\vec{x},\vec{r}) \quad (2)$$

The weight function $w_Q(\boldsymbol{\xi},\mathbf{x})$ includes terms for obliquity and geometrical spreading compensation. For completeness, it should also normalize for the direction of illumination provided by elements of $U(\boldsymbol{\xi},\tau_D)$ to the image point $I(\mathbf{x})$. The need for this normalization is a consequence of choosing the acquisition geometry as the basis of integration; the integrand of equation 1 describes the illumination of an image point at **x**, but the variable of integration does not evenly sample the illumination angle.

Implementation

The direction of illumination **q** can be expressed as the sum of the source and receiver slowness vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ (Figure 1). This vector quantity is a function of the overburden velocity structure. The zero-offset raypath cartoon in Figure 2 illustrates that, given regular acquisition geometry, the illumination vector **q** at an image point will not vary regularly in the general case. Without proper weighting compensation, this irregularity in illumination angle will produce artifacts in the migrated image.

Operto et al. (1998) made the observation that the illumination normalization issue is significantly simplified if Kirchhoff migration is reformulated as summation over the illumination variable. The expression for the weighted diffraction stack is then

$$I(\vec{x}) = \frac{-1}{2\pi}\iint dq_1 dq_2\ w(\vec{\xi},\vec{x})\frac{d}{dt}(U(\vec{\xi},\tau_D(\vec{\xi},\vec{x}))). \quad (3)$$

Here $w(\boldsymbol{\xi},\mathbf{x})$ includes only terms for obliquity and geometrical spreading.

By making the illumination angle the basis of integration, equation 3 implicitly guarantees illumination normalization. Koren et al. (2002) have shown imaging examples explicitly implementing this approach as an output-driven migration in which rays are shot from the image point to the surface. In general, however, output-driven migration schemes, in which travel time tables are calculated with respect to the image points in the output *volume*, are significantly more expensive than input-driven migration schemes, in which travel time tables are calculated with respect to locations on the measurement *surface*.

If the practical problems associated with output-driven migration are to be avoided, a Jacobian function is needed to reconcile w_Q in equation 1 with w in equation 3 so that illumination angle compensation may be included in an input-driven migration procedure. This change in variables is accomplished using the Beylkin determinate as described in Jousset et al. (2000). Given that this function relates the acquisition coordinates $\boldsymbol{\xi}$ to the direction of illumination **q**, it is a function of both the acquisition geometry and the subsurface model.

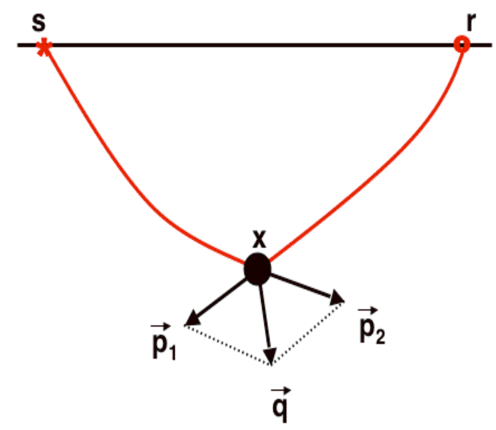

Figure 1: The illumination vector **q** is the sum of the slowness vectors $\mathbf{p_1}$ and $\mathbf{p_2}$. The slowness vectors have equal length (the inverse of the velocity at **x**), so **q** bisects the angle between them.

The vector **q** is a function of the overburden velocity structure, and, in the general case, it cannot be computed until source and receiver travel time functions are combined for imaging. For a general input-driven migration implementation, however, illumination angle compensation requires knowledge of the final distribution of all illumination angles at each image point (Albertin et al., 1999). Fortunately, it is possible to side-step this onerous implementation requirement if some form of acquisition regularization has been applied to the survey data before migration. Illumination angle compensation may then be calculated using derivatives of the travel time tables (Kästner, 2001).

Discussion

Illumination angle normalization is effectively a contribution scaling, which is a requirement for amplitude preservation. For constant-azimuth common-offset seismic data, the illumination angle term is a function of the acquisition midpoint density and the density and orientation of rays at each reflection point.

In practice, illumination angle compensation suppresses Kirchhoff migration artifacts due to focusing of the wavefield by velocity anomalies in the overburden. Structures such as salt stocks tend to funnel rays traveling through salt bodies into relatively narrow packages; energy from these narrow windows of illumination penetration are smeared throughout the subsalt region when normalization for illumination angle is neglected.

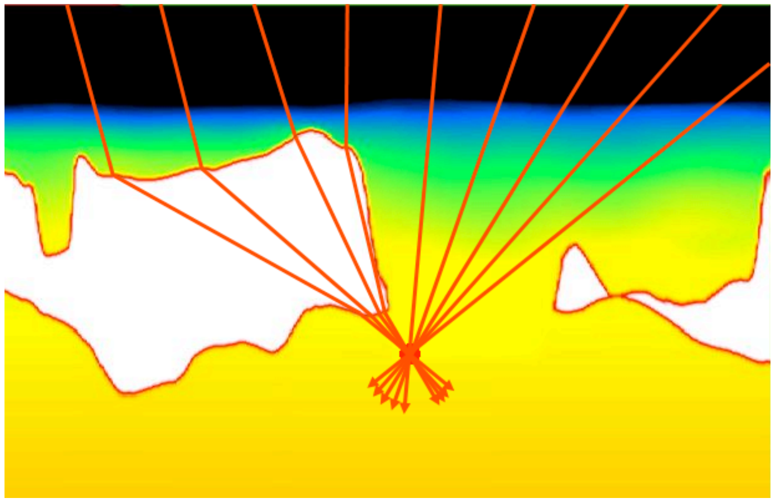

Figure 2: The effect of salt on the illumination vector **q** at an image point for a regular zero-offset acquisition geometry.

Figures 3 and 4 show images from two 3-D Kirchhoff prestack depth migrations of a deep water salt body. The same velocity model was used for both migrations. The migration that produced the image in Figure 3 was run without illumination angle compensation. Note the typical Kirchhoff migration artifacts where indicated. The image shown in Figure 4 was produced using Kirchhoff prestack depth migration with illumination angle compensation. Subsalt migration artifacts that are present in Figure 3 are greatly attenuated.

Summary

Kirchhoff and one-way wave equation prestack depth migration methods share a common foundation in migration/inversion theory. However, inherent differences in algorithm methodologies pose different challenges when implementing each method. In the case of illumination normalization, extra effort must be extended in both algorithm design and computational expenditure to include amplitude corrections that are implicitly included in wave equation formulations. In current Kirchhoff migration practice, illumination angle compensation is frequently neglected to avoid this additional effort.

The example presented with this paper demonstrates that explicit inclusion of illumination angle compensation in Kirchhoff prestack depth migration produces imaging results that are significantly improved over those that are produced by Kirchhoff prestack depth migration without this normalization. This is particularly true in the case of subsalt depth imaging, where illumination distortions due to the salt geometry are most severe.

References

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Acknowledgements
The author benefited greatly in the preparation of this paper from the work and insights expressed by Uwe Kästner in an internal Geotrace/Ensign report (2001). Scott Boyer of Geotrace Technologies produced the example presented in this paper using migration software created by Uwe Kästner and Stefan Buske.

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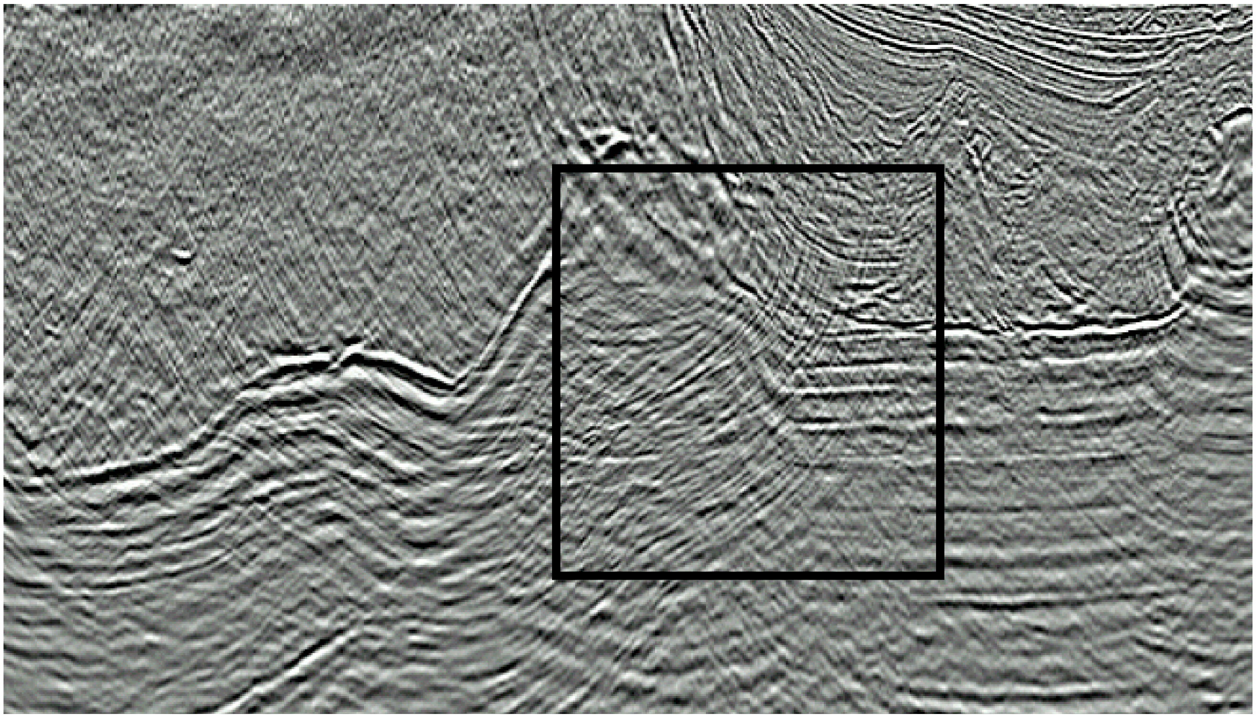

Figure 3: Kirchhoff prestack depth migration of a deep water salt body without illumination angle compensation. Note the subsalt migration artifacts where indicated. Data provided courtesy of Unocal.

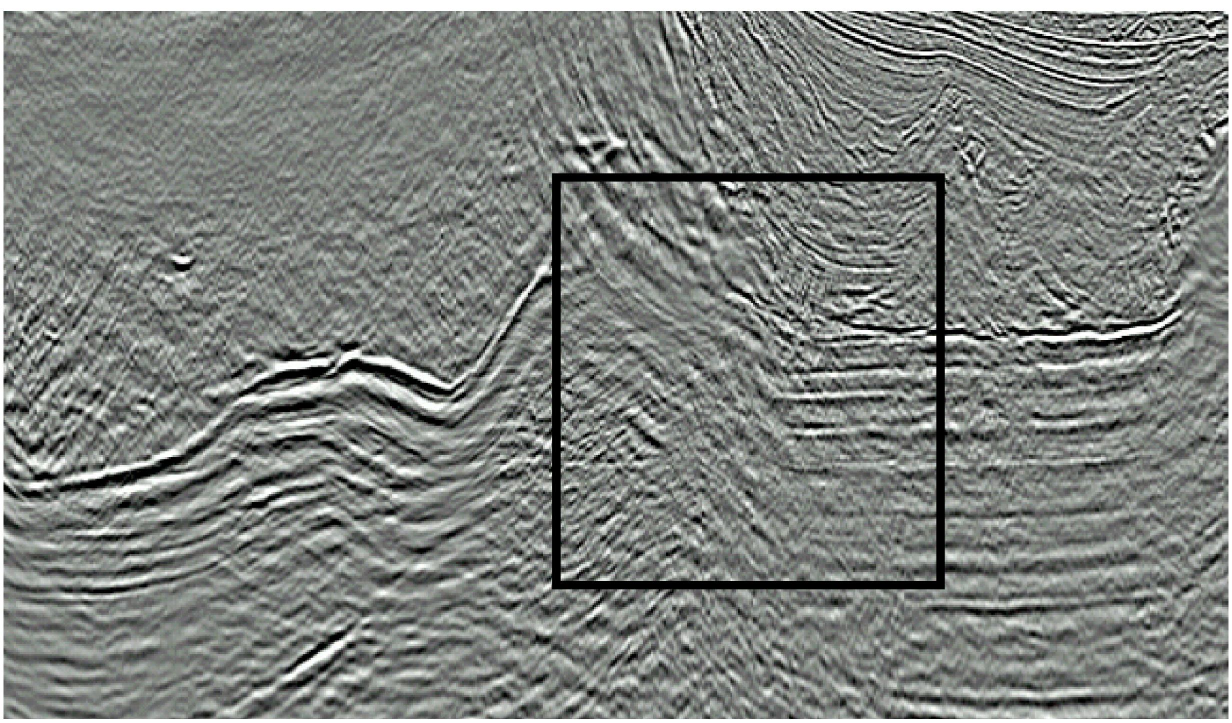

Figure 4: Kirchhoff prestack depth migration of a deep water salt body with illumination angle compensation. Subsalt migration artifacts present in Figure 3 are greatly attenuated. Data provided courtesy of Unocal.